iv . From the two terms of "$s.\sin\alpha_{12}$" and "$s.\cos\alpha_{12}$", the values of s and α_{12} can easily be computed.

v . From the value of $d\alpha$, obtained in step (i) and the value of α_{12}, the value of α_{21} can be computed.

7.2 Review of Spherical Trigonometric Formulae:

Recalling the successive steps to be followed for the construction of a triangulation network, we conclude that a knowledge of "Spherical Trigonometry" is a must. These steps are summarized here as:

1. Reconnaissance and choice of the location of stations.
2. Field measurements of, at least one base line and a suitable number of angles more than the necessary number.
3. Choice of an initial point, and carring out the required astronomical observations for a determination of the coordinates of the initial point as well as the azimuth of one line, usually the base line.
4. Reduction of the field measurements to the computational surface, the reference sphere.
5. Least-square adjustment of the reduced angular measurements.

6. Computations of the length of sides using the reduced length of the base line, step (4), as well as the adjusted values of the angles, step (5).
7. Computation of the geodetic positions, coordinates, of the triangulation stations (using the direct geodetic problem formulae).

Thus, in order to transform the reduced, and adjusted, field measurements into a useful information on the length of the sides of the triangulation network as well as their mutual azimuths, we mention, without proof, the following spherical trigonometric formulae; Fig. (7.2):

1) In case of being given two angles and a side opposite one of them, as A, B, and a, use is made of the SINE RULE as follows:

$$\sin b = \frac{\sin a \sin B}{\sin A} \tag{7.3}$$

C, the third angle, is obtained as in case (6) following and then c by substituting c and C for b and B in (7.3). As in the plane trigonometry, the value of b is ambiguous in this case, since there are two possible values fot it, corresponding to the computed value of one being the supplement of the other. The fact that the greater angle subtends the greater side removes the ambiguity in certain cases, and in practice it is usually possible to select the correct value for the unknown angle .

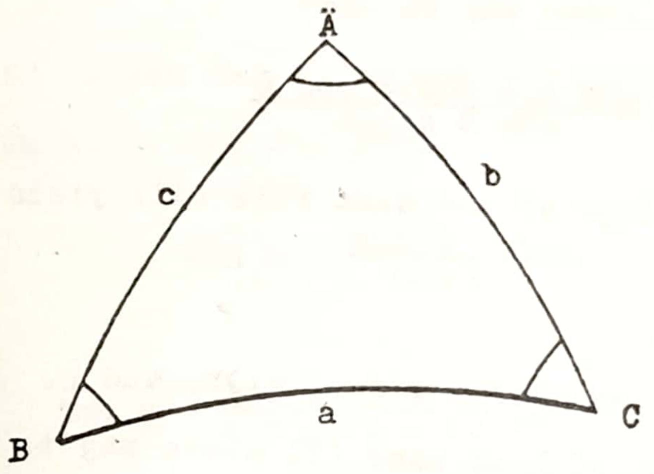

Fig. (7.2): The General Spherical Triangle, Formed of Six Elements: 3 angles, and 3 sides.

2) Given two sides and the subtended angle as b, c and A; then we may use the COSINE RULE

$$\cos a = \cos b \cos c + \sin b \sin c \cos A , \qquad (7.4)$$

B and C can then be obtained according to case (6), given below.

Formulae (7.3) and (7.4) are the fundamental formulae of the spherical Trigonometry, as they represent the basis on which all of the other formulae can be derived.

3) Given the three angles A, B, and C. Then the COSINE RULE for sides may be used

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C} \qquad (7.5)$$

and the application of the sine rule will yield both b and c.

4) Given the three sides, a, b, and c. In this case, the COSINE RULE of case (2) above may be used when rewritten as:

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \qquad (7.6)$$

and the other two angles are obtained by applying the sine rule.

5) Given two angles and the side between them, as A, B, and c. Then we may use

$$\tan \tfrac{1}{2}(a + b) = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} \tan \frac{c}{2} , \qquad (7.7)$$

$$\tan \tfrac{1}{2}(a - b) = \frac{\sin \frac{1}{2}(A - B)}{\sin \frac{1}{2}(A + B)} \tan \frac{c}{2} , \qquad (7.8)$$

whence $\frac{1}{2}(a + b)$ and $\frac{1}{2}(a - b)$, and thence

$$a = \tfrac{1}{2}(a + b) + \tfrac{1}{2}(a - b), \qquad (7.9)$$

and

$$b = \tfrac{1}{2}(a + b) - \tfrac{1}{2}(a - b). \qquad (7.10)$$

The third angle can then be obtained from the Sine Rule.

6) Given two sides and the angle opposite one of them, as a, b, and A. Then:

$$\sin B = \frac{\sin A \sin b}{\sin a} \qquad (7.11)$$

$$\tan \frac{c}{2} = \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} \tan \tfrac{1}{2}(a - b) \qquad (7.12)$$

after which C can be found by substituting C and c for B and b in (7.11).

As in case (1) above, there may be an ambiguity about B and c as computed from (7.11) and (7.12), but as before, it is usually not difficult to decide which value to adopt.

7) The following two formulae are sometimes useful:

$$\sin a \cos B = \sin c \cos b - \cos c \sin b \cos A \qquad (7.13)$$

$$\sin A \cos b = \sin C \cos B + \cos C \sin B \cos a \qquad (7.14)$$

Similar formulae, involving c and C on the left-hand side, are, of course, obtained by making the appropriate substitution.

8) <u>The Right-angle Triangle</u>:

In right-angled triangles there are <u>ten</u> formulae dealing with the <u>five</u> unknown elements (three sides and two angles; one angle being a right angle). These can be remmbered by means of "<u>Napier's five-parts circle for right-angled triangles</u>", which is a mneomonic in the following terms:

i . Divide a circle into five parts, Fig. (7.3).

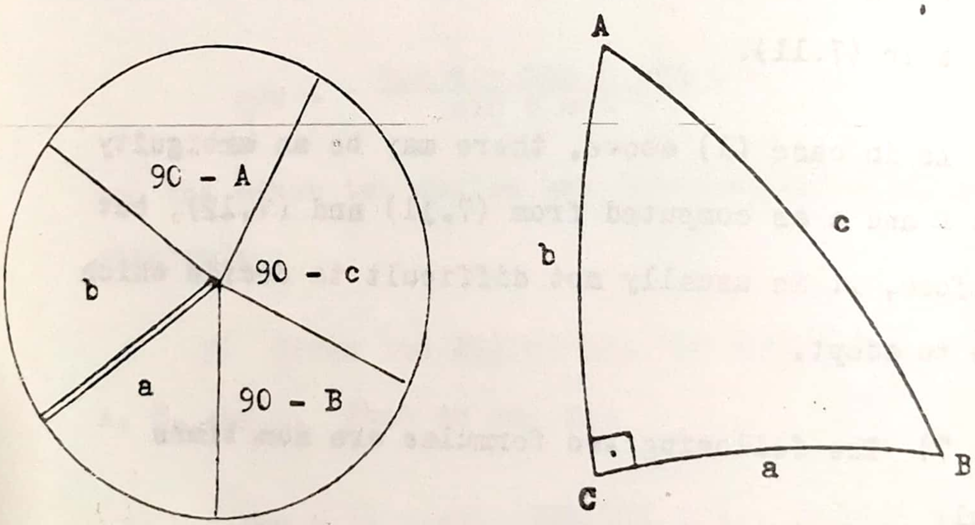

Fig. (7.3) Right-angled Triangle, and Napier's Five-Parts Circle.

ii . Insert in order the five parts of the triangle, (the sixth, C the right angle, being omitted), beginning with the two sides enclosing the right angle, and continuing with the "COMPLEMENTS" of the remaining three parts.

iii. Then, Napier's rule is:

sin(middle part)= Product of tan(ajacents) ,

= Product of cos(opposites) .

(7.15)

Note that the vowels in each i entity of (7.15) are the same, i.e.

s<u>I</u>n m<u>I</u>ddle = Product of t<u>A</u>n <u>A</u>djacents,

= Product of c<u>O</u>s <u>O</u>pposites.

7.3 Some Practical Approximations:

In the practical computation of a triangulation network, the use of spherical triangles is commonly possible, and, the solution of few spherical triangles is not a difficult task. However, the direct solution of a series of spherical triangles in an extended triangulation system would be extremely complicated. On the other hand, the solution of a series of plane triangles is quite simple. Only the plane law of sines